The Riemann Zeta-function $\zeta(s)$

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The Riemann Zeta-function is defined as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

for complex values of s. While converging only for complex numbers s with $\Re(s) > 1$, this function can be analytically continued to the whole complex plane (with a single pole at $s = 1$). The Riemann Zeta-function was first introduced by Euler with the computation of

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

but it was Riemann who, in the 1850's, generalized its use and showed that the distribution of primes is related to the location of the zeros of $\zeta(s)$. Riemann conjectured that the non trivial zeros of $\zeta(s)$ are located on the *critical line* $\Re(s) = 1/2$. This conjecture, known as the *Riemann hypothesis*, has never been proved or disproved, and is probably the most important unsolved problem in mathematics. The Riemann hypothesis makes the zeta function so famous, and numerical computation have been made to check it for various sets of zeros.

We expose here the most classical results about the zeta-function, together with some computational aspects. Our presentation is divided into several parts that are listed here :

- *The Riemann Zeta-function $\zeta(s)$: generalities*

- *Numerical evaluation of the Riemann Zeta-function*

- *Distribution of the zeros of the Riemann Zeta function*

- In preparation : Computation of zeros of the Riemann Zeta-function.

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[1]This pages are from *//numbers.computation.free.fr/Constants/constants.html*